The course “Physics, Mathematic”
Tasks for the examination

1. Find the derivative of the function $y = \cos(x^2 + 3)$.

2. Find the derivative of the function $y = 3x^4 - e^x + 6\sqrt[5]{x^3}$.

3. Find the derivative of the function $y = \sin(2x) \cdot 4^x + \ln 3$.

4. There is the equation of motion for a body: $S = 2t^3 + 3t^2 + 2$. Find the speed and acceleration of the body after 2 seconds.

5. Quantities of a substance produced in a chemical reaction changes over time according to this law: $Q(t) = 4(1 + 8e^{-16t})$. Find the rate of this chemical reaction.

6. A pathological process develops over time according to this equation: $y = t^3 - t^2 + 1$. Find the speed of this process after 2 seconds.

7. Find the differential *dy* of the given function: $y = \ln x \cdot arctg\, x$.

8. Find the differential *dy* of the given function: $y = \ln(x^3 + 1)$.

9. Find the increment of the function $y = (3x^2 - 2)^3$ for x belonged to the interval from 1 to 1.001.

10. Evaluate the definite integral: $\int_0^{\pi} \sin x \cdot \cos^3 x \, dx$.

11. Find the integral: $\int \frac{\cos^3 x - 1}{\cos^2 x} dx$.

12. Find the integral: $\int \frac{49 - x^2}{x + 7} dx$.

13. Calculate the volume of a body, formed by rotating the figure around the axis *Ox* bounded by the lines $y = x + 4$, $y = 0$, $x = 0$, $x = 6$.

14. Find the work of the variable force $f(x) = e^{2x}$ for a linear displacement of a material point from the position $x_1 = 0$ to the position $x_2 = 3$.

15. Solve the equation: $y' = x \cdot y$.

16. Find the solution of the equation $y' = y/(x + 1)$ under the condition $y(2) = 6$.

17. The rate of growth of a bacterial culture is equal to 0.1 for the bacterial population x(t). Describe this process by using the differential equation. Solve this equation under the condition $x(0) = 1000$.

18. A student knows 20 questions from 25 questions of a test. A question sheet consist of 3 questions. Find the probability that the student doesn't know at the minimum 1 question from the question sheet.

19. When 1000 glass vases is transported, the probability of breaking 1 vase is equal to 0.002. Find the probability of breaking 4 vases during transportation.

20. A batch of 1000 ampoules of novocaine contains 400 ampoules from the first factory, 350 ampoules from the second factory, and 250 ampoules from the third factory. The probabilities of defective ampoules produced by three factories are equal to 0.25, 0.2, and 0.15, respectively. Find the probability that one randomly selected ampoule from this batch is not defective.

21. The random variable X is normally distributed with a mean of 25 and standard deviation of 4. Find the probability $P(0 < X < -5)$.

22. The random variable X is normally distributed with a mean of 2 and standard deviation of 4. Find the distribution function $F(x)$ and probability density function $f(x)$.

23. A sample of calls to an ambulance station in 15 minutes is following: 1, 4, 2, 3, 2, 3, 3, 2, 3, 4. Construct the variation series and discrete series (table of frequency distributions). Plot the frequency polygon.

24. The data follow: 12, 10, 17, 13, 20, 18, 25, 27, 24, 30. Find the main characteristics of this sample.

25. Measurements of patients' heart rate were obtained in a hospital. A sample of nine patients resulted in the following data: 71, 70, 74, 70, 72, 71, 70, 73, 72. Construct the discrete series (frequency table). Find the statistical characteristics of the sample.

26. Measurements of respiratory rate of patients were recorded as follows: 12, 14, 12, 15. Write down the variation series. Find the mean and variance.

27. Diameters of human erythrocytes were measured using a microscope. The following sample was obtained (in micrometers): 5, 8, 11, 8. Find a 95% confidence interval of the mean.

28. Retinal vascular permeability was investigated. The data collected are as follows: 14, 12, 16, 11, 15, 17, 13, 15, 16, 11. We assume that the data is normally distributed with standard deviation $\sigma = 5$. Find a 95% confidence interval of the mean.

29. A study of mental development in school-age children was carried out. The statistical results for $x =$ *class* 1 and $y =$ *class* 2 are given in Table 1. Consider testing H_0: $\mu_x = \mu_y$ (the two population means are equal) by using the unpaired t-test at level of significance 0.05.

Table 1

Sample sizes		Means		Variances	
$n_y = 20$	$n_y = 10$	$\overline{x} = 29.233$	$\overline{y} = 27.562$	$s_x^2 = 5.62$	$s_y^2 = 2.19$

30. A study was carried out to compare a treatment effect for following two medicines: $x =$ *proprietary aerosol drug* in a control group ($n_x = 8$) and $y =$ *experimental aerosol drug* in an experimental group ($n_y = 12$). The results of the

study are given in Table 2. Consider testing H_0: $\sigma^2_x = \sigma^2_y$ (the population variances in the two groups are equal) by using the F-test at level 0.05.

Table 2

Sample sizes		Variances	
$n_x = 8$	$n_y = 12$	$s_x^2 = 0.18$ gram^2	$s_y^2 = 0.04$ gram^2

31. Consider the following data (Table 3) on y = *animal height* (cm) and x = *animal weig*ht. Calculate the correlation coefficient.

Table 3

x_i	40	41	42	43	44	45
y_i	15	20	25	29.6	36.4	38.8

32. The dependence between the men's thorax volume x (cm^3) and their height y (cm) was studied. The results of the observations are given in Table 4. Find the simple linear regression model.

Table 4

x_i	83	87	91	94	105
y_i	170	176	181	179	184

33. The angle between the incident beam and the reflected beam is 30°. What is the angle between the incident beam and the mirror surface?

34. The average concentration of K^+ ions in the axoplasm of a squid giant axon is equal to 410 mol/m^3. The average concentration of K^+ ions in seawater is equal to 10 mol/m^3. Calculate the Nernst potential at 27 °C.

35. An 60-kg body absorbed an energy of 1 J within 6 hours. Find the absorbed dose and absorbed dose rate.

36. The half-life of ${}^{30}_{15}P$ is 3 minutes. Find the decay constant of ${}^{30}_{15}P$.

37. A malignant tumor accumulated radioactive boron ${}^{10}_{5}B$ was irradiated with a beam of neutrons. As a result, ${}^{7}_{3}Li$ and some ionizing radiation were formed. This ionizing radiation affected on the tumor. What is the type of this ionizing radiation?

38. Determine the flux of bremsstrahlung X-rays for an X-ray tube with applied voltage of 10 kV, if its anode is made of tungsten (atomic number $Z = 74$), the current in the cathode is 1 A. The proportionality coefficient k is 10^{-9} V^{-1}.

39. There is a spring with a coefficient of elasticity, k, of 200 N/m. A force stretching the spring by x meters is described by the following equation: $F(x) = kx$. Find the work to stretch the spring from the equilibrium position to the position of 0.1 meter.

40. The Doppler effect using ultrasonic waves of frequency $2.25 \cdot 10^6$ Hz is used to monitor the heartbeat of a fetus. A (maximum) beat frequency of 240 Hz is observed. Assuming that the speed of sound in tissue is 1540 m/s calculate the maximum velocity of the surface of the beating heart.

41. Determine the phase difference in the pulse wave between two points of an artery located at a distance of 20 cm from each other. The speed of the pulse wave is equal to 10 m/s, the heart oscillations are harmonic with a frequency of 1.2 Hz.

42. A microscope uses an eyepiece with a focal length of 1.70 cm. Using a normal eye with a final image at infinity, the barrel length is 17.5 cm and the focal length of the objective lens is 0.65 cm. What is the magnification of the microscope?

43. The sound intensity is 10^{-8} W/m2 at a 1000-Hz frequency. Find the loudness of this sound.

44. An ultrasound pulse passes through the abdomen, reflecting from surfaces in its path. The speed of sound waves in human tissue is 1540 m/s. The time of the wave propagation is 320 μs. What is the distance from the transducer to the vertebra?

45. The femur bone in the human leg has a diameter of 30 mm and wall-thickness of 3 mm. How much compressive force can it withstand before breaking?

46. There is the human femur. A strain is 0.025 for a stress of 5 Pa, and a strain is 0.055 for a stress of 11 Pa. How does the elasticity modulus of the femur change?

47. Determine the change in length of 4-cm tendons with a diameter of 6 mm under a force of 31.4 N. The elastic modulus of the tendons is equal to 10^9 Pa.

48. Find the maximum volume of blood passing through an aorta during 1 second to keep laminar flow. An aortic diameter is 2 cm, a blood viscosity is 5 mPa·s.

49. The pulse wave in an artery has a speed of 8 m/s. It is known that the ratio of the artery radius to the thickness of the artery wall is 6, and a density of the artery wall is 1.15 g/cm^3. What is the elasticity modulus for this artery?

50. Determine the Reynolds number in a vessel with a diameter of 3 mm. A blood velocity in this vessel is 1.8 m/s, a blood density is 1600 kg/m^3, a blood viscosity is $5 \cdot 10^3$ Pa·s. Is the turbulent blood flow?

51. A 20-cm thickness concrete reduces the intensity of a beam of ionizing radiation by 16.5 times. Find the linear attenuation factor. Find the thickness of the concrete reducing the radiation intensity by 2 times.

52. A doctor faultily performed X-ray therapy for a patient. A radiation source was placed at a distance of 30 cm instead of 40 cm. Find the exceeding of the exposure dose of radiation for the patient.

53. A 20-cm thickness concrete reduces the intensity of a beam of ionizing radiation by 16.5 times. Find the linear attenuation factor. Find the thickness of the concrete reducing the intensity of ionizing radiation by 2 times.

54. A vessel passes 60 ml of blood during a 0.25-s systole. A radius of this vessel is 1.5 cm. Find the average linear velocity of blood.